LETTERS TO THE EDITOR

To the Editor:

The purpose of this letter is to comment on the mass-transfer model used by Zhang et al.[1] in their article entitled "Modeling of CO_2 mass transport across a hollow fiber membrane reactor filled with immobilized enzyme". To model mass transfer in the lumen side of a hollow-fiber, the authors explicitly assume that (1) diffusion along the axial direction is negligible, and (2) gas flow inside the hollow fiber is laminar and incompressible. Under these assumptions, the equation of continuity of CO_2 reads at steady state

$$v_z \frac{\partial C}{\partial z} = D_g \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) \quad (1)$$

where $v_z = 2U[1-(r/R)^2]$ is the parabolic velocity distribution, and U is the average flow velocity. Equation 1 is subject to the following boundary conditions[1]

$$C = C_i \text{ at } z = 0 \text{ and all } r, \quad (2)$$

$$\frac{\partial C}{\partial r} = 0 \text{ at } r = 0 \text{ and all } z, \quad (3)$$

$$-D_g \frac{\partial C}{\partial r} = K_{OV}(C - C_e) \text{ at } r = R_f \text{ and all } z \quad (4)$$

In terms of dimensionless variables $C^* \equiv (C - C_e)/(C_i - C_e)$, $r^* \equiv r/R_f$ and $z^* \equiv zD_g/.UR_f^2$, the solution given by Zhang et al.[1] may be written as follows

$$C^*(r^*, z^*) = \sum_{n=1}^{\infty} \frac{2J_0(\lambda_n r^*)\exp(-\lambda_n^2 z^*)}{\lambda_n\left[1+\left(\frac{D_g\lambda_n}{K_{OV}R_f}\right)^2\right]J_1(\lambda_n)} \quad (5)$$

where the eigenvalues λ_n, are roots of the following equation

$$\lambda_n D_g J_1(\lambda_n) = K_{OV} R_f J_0(\lambda_n). \quad (6)$$

We would like to point out that Eq. 5 is inconsistent with the laminar flow assumption, which mandates a parabolic velocity distribution for steady flow of a Newtonian fluid. Rather, Eq. 5 given by Zhang et al.[1] satisfies Eq. 1, and the aforementioned stated boundary conditions if and only if $v_z = U$, i.e., for a uniform velocity distribution. The correct solution of Eq. 1 under the assumptions explicitly made by Zhang et al.[1] is not Eq. 5, but a solution to the well-known Graetz problem,[2] which is expressed in terms of Kummer's confluent hypergeometric function $M(a, b, x)$, as follows[3]

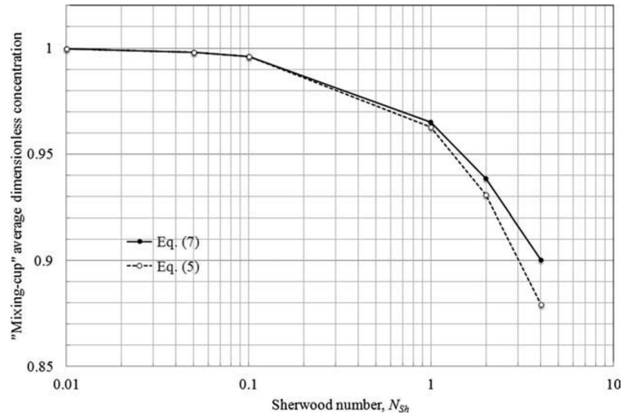

Figure 1. 'Mixing-cup' average concentration (dimensionless) as a function of N_{sh} for $N_{Gz} = 100$.

$$C^*(r^*, z^*) = \sum_{n=1}^{\infty} \zeta_n M\left(\frac{1}{2} - \frac{\chi_n}{4}, 1, \chi_n r^{*2}\right)\exp\left(-\frac{\chi_n r^{*2}}{2}\right)\exp\left(-2\chi_n^2 z^*\right) \quad (7)$$

where the constants ζ_n are given by

$$\zeta_n = \frac{\int_0^1 M\left(\frac{1}{2} - \frac{\chi_n}{4}, 1, \chi_n r^{*2}\right)\exp\left(-\frac{\chi_n r^{*2}}{2}\right) r^*(1 - r^{*2})dr^*}{\int_0^1 \left[M\left(\frac{1}{2} - \frac{\chi_n}{4}, 1, \chi_n r^{*2}\right)\exp\left(-\frac{\chi_n r^{*2}}{2}\right)\right]^2 r^*(1 - r^{*2})dr^*} \quad (8)$$

the eigenvalues, χ_n, are roots of the following equation

$$\left(\frac{N_{Sh}}{2} - \chi_n\right) M\left(\frac{1}{2} - \frac{\chi_n}{4}, 1, \chi_n\right) + 2\chi_n\left(\frac{1}{2} - \frac{\chi_n}{4}\right) M\left(\frac{3}{2} - \frac{\chi_n}{4}, 2, \chi_n\right) = 0 \quad (9)$$

and $N_{Sh} \equiv 2K_{OV}R_f/D_g$ denotes the Sherwood number. Note that the solution to Eq. 1 subject to the aforementioned stated boundary conditions is determined completely by the values of the Sherwood and Graetz numbers, the latter defined as $N_{Gz} \equiv 4UR_f^2 \big/ zD_g$. The difference between Eq. 5 and Eq. 7 can be significant for sufficiently large values of N_{Sh} and N_{Gz}. This is shown in Figure 1, which plots the "mixing-cup" average concentration for $N_{Gz} = 100$ as a function of N_{sh}. Unfortunately, Zhang et al.[1] did not report values of the overall mass-transfer coefficient K_{OV}, so it is difficult to determine the extent to which conclusions drawn about the specific system they modeled may have been affected by their neglect of the velocity distribution.

Notation

- C = CO_2 concentration, mol/m^3
- C_e = inlet CO_2 concentration in the gas feed fiber, mol/m^3
- C_i = outlet CO_2 concentration from the sweeper gas fiber, mol/m^3
- D_g = diffusion coefficient, m^2/s
- K_{OV} = overall mass-transfer coefficient, m/s
- r = radius, m
- R_f = fiber radius, m
- U = average flow velocity, m/s
- z = axial position, m

Literature Cited

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DOI 10.1002/aic.13769
Published online February 13, 2012 in Wiley Online Library (wileyonlinelibrary.com).